NUMERICAL MODELLING
OF
TSUNAMI WAVE EQUATIONS

A
thesis

submitted in partial fulfillment of the requirements for the award of the degree of

MASTER OF SCIENCE
IN
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submitted by
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ROURKELA

DECLARATION

I hereby declare that the work which is being presented in the report entitled " **NUMERICAL MODELLING OF TSUNAMI WAVE EQUATIONS** " in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela is an authentic record of my own work carried out under the supervision of Prof. S. Chakraverty.

The matter embodied in this has not been submitted by me for the award of any other degree.

May, 2014

(Hari Shankar Shaw)

CERTIFICATE

This is to certify that the project report entitled **" NUMERICAL MODELLING OF TSUNAMI WAVE EQUATIONS "** submitted by **Hari Shankar Shaw** to the National Institute of Technology Rourkela, Odisha for the partial fulfillment of requirements for the degree of master of science in Mathematics is a bonafide record of review work carried out by him under my supervision and guidance. The contents of this report, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

May, 2014

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ABSTRACT

This report investigates the modelling of tsunami wave using one dimensional shallow water equations (SWEs) by numerical methods namely finite difference method (FDM) and finite volume method (FVM). We have used one dimensional SWEs to model the water wave propagation i.e. we study the variation of water surface elevation with finite distance. We obtained the SWEs from Euler's equation of mass and momentum assuming a long wave approximation. First of all we approximate the SWEs using FDM and then by FVM for showing the behaviour of water surface elevation with distance. After approximating the SWEs using both the numerical method, results have been shown using different schemes viz. FDM as well as FVM. Moreover, in actual practice, we may have incomplete information about the variables being a result of errors in modelling, observations, or by applying different initial as well as boundary conditions etc. Rather than the particular value of water surface we may have only the bounds of the values. These bounds may be given in term of interval. Thus we have developed interval finite volume method (IFVM) also for approximating one dimensional SWEs to model tsunami wave with uncertain (interval) parameter. Next, numerical results have been shown using IFVM. Then a comparision study has been investigated to compare the results of both the method i.e FDM and FVM. Finally all computed results are shown in terms of tables and plots.

Contents

1 Intoduction

Tsunamis are generated by the movement of sea bottom due to long waves of earthquakes. The impulsive sea floor movement in the earthquakes region causes the water surface region instantaneously as discussed in [1]. The sudden gain in potential energy converts to kinetic energy by the gravitational force which serves as the restoring force of the system. Generally, tsunamis are treated as shallow water waves.

Tsunami is a Japanese word that is the combination of two words :“ tsu ” means harbor and “ nami ” means wave. Therefore, tsunami literally means “harbor waves”. The word was originally created to describe large amplitude oscillations in a harbor under the resonance condition given in [2]. The most common cause of tsunami are under sea earthquakes.

In this report we have used shallow water equations (SWEs) to model water wave propagation in one dimension. We obtain the SWEs from Euler's equation of mass and momentum considering a long wave approximations given in [1], [3]. SWEs state the propagation of water waves whose wave length is much longer than the depth of water. Therefore, we have modeled tsunami wave using SWEs.

Shallow Water Equations (SWEs)

Shallow Water Equations (SWEs) are a system of hyperbolic partial differential equations (PDEs) governing the flow of fluid in the rivers, channels, oceans and costal regions. We have investigated SWEs from mass and energy conservation principle expressed in the Navier-Stokes equations. SWEs give the idea about the flow of water waves, especially those water wave whose wave length is much longer than the depth (basin) of water.
The wavelength of tsunami waves are far longer than the normal waves. A tsunami wave initially resembles a rapidly rising tides for this reason they are often referred to as tidal waves. The average depth [3] of ocean nearly 5 Km, which is compared with the wavelength long waves or tsunamis, which may exceed 100 Km.

Although the impact of tsunamis is limited to coastal areas, their destructive power can be enormous and they can affect entire ocean basins; in 2004 Indian Ocean tsunami was among the deadliest natural disasters in human history with over 230,000 people killed in 14 countries bordering the Indian ocean. One can get SWEs by neglecting the bottom friction and assuming long wave approximations from the Euler equations of mass and momentum.

SWEs are used to model Dam Beaks, strom surges, solute transport, river flows, economical model etc given in [3]. The problem with SWEs is, they are difficult to model in dry areas where water is not present. SWEs are only defined in wet regions. Thus for these type of equations we need actually to deal with moving boundary problems.

How do SWEs arise ?

SWEs are investigated by Navier-Stokes (N-S) equations, which describe the motion of fluid. Also, the N-S equations are derived from the equations of conservation of mass and linear momentum discussed by Imamura [1]. So, from the translation motion of fluid element and neglecting the vertical acceleration, the equations of mass conservation and momentum in one dimensional problem are described as follows [1]

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \\ \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + (1/\rho)\frac{\partial P}{\partial x} = 0 \\ \\ \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + g + (1/\rho)\frac{\partial P}{\partial z} = 0 \end{cases} \tag{1}$$

where, x is the horizontal axis and z is the vertical axis, t is time, η is the water surface elevation, h is basin depth, u, w are the velocities of fluid in the x and z directions respectively, and g is the acceleration due to gravity. Finally after taking the dynamics and kinetic conditions at surface and bottom [3], we get the Shallow Water Equations (SWEs).

Mathematical modeling plays a vital role in the area of tsunami science, such as in the area of scientific studies for tsunami propagation and initiation. This thesis investigates the numerical solution of one dimensional SWEs using numerical methods namely Finite Difference Method (FDM) and Finite Volume Method (FVM).

FVM is one of the most useful method for modeling the SWEs, long waves, radiative transfer, etc. FVM is widely used in engineering, fluid mechanics, petroleum engineering, computational fluid dynamics, heat and mass transfer, etc. The most important feature of this method is that numerical flux is conserved from one discretization cell to its neighbor. This feature makes FVM quite attractive for modeling problems in fluid mechanics, heat transfer and semi conductor device simulation.

FVM is a method of representing and evaluating the partial differential equations to algebraic equations. In this method we calculate the values at discrete places on meshed geometry as in finite difference method (FDM) or finite element method (FEM). Finite (control) refers to small volume surrounding each node point on a mesh. Then we used interval finite volume method (IFVM) to the SWEs and have investigated the variation of Tsunami wave.

The shallow water equations introduced in [6] is very commonly used for numerical solution. Modelling of tsunami wave has been solved using linear Leap-frog method by Imamura, Yalciner [1]. A detailed study of SWEs is given in IUGG/IOC time project [3] for numerical methods in tsunami simulation. A numerical modelling on one dimensional and two dimensional SWEs using FDM discussed in Junbo Park [4] and added the numerical simulation of wave propagation. A lot of research work on one and two dimensional convection-diffusion problems, [8] has been solved using finite volume scheme by Versteeg and Malalasekera. Cebeci, et al. [9] proposed real life problems using FVM.

In this report our main aim is to develop an efficient numerical method for solving SWEs to model Tsunami wave. Generally, the values of variables or properties are taken as crisp but in actual case the accurate (crisp) values may not be obtained. To overcome the vagueness we use interval in place of crisp values. So, next aim is to study the interval finite volume method (IFVM). The IFVM has been developed here to study the variation of tsunami waves.

In this report we first discuss the introduction of tsunami wave and shallow water waves and their origins in section 1 and 2. A detailed study of one dimensional SWEs using different schemes of FDM has been done in section 3. Then we have solved one dimensional SWEs using FVM with different schemes of interpolation and variation for FVM and IFVM in section 4. Section 5 deals with the numerical results for one dimensional SWEs using FDM and FVM . A comparative numerical results using FDM and FVM has been investigated in section 6. Also conclusion and future work has been included in section 6 and finally references are cited.

2 One dimensional shallow water equations (SWEs)

The shallow water Eqs. in one dimension is given by [1]

$$\begin{cases} \dfrac{\partial \eta}{\partial t} + \dfrac{\partial M}{\partial x} = 0 & \text{(mass conservation law)} \\ \\ \dfrac{\partial M}{\partial t} + gD\dfrac{\partial \eta}{\partial t} = 0 & \text{(momentum conservation law)} \end{cases} \tag{2}$$

where,

$\eta =$ Water surface elevation

$M =$ Discharge flux in the positive x-direction

$g =$ Acceleration due to gravity

$D =$ Total thickness of water

$h =$ Basin depth of water

Thus, $D = \eta + h$

Fig. 1 shows the behaviour of one dimensional shallow water equations, where vertical axis represents water surface elevation and horizontal axis represents distance.

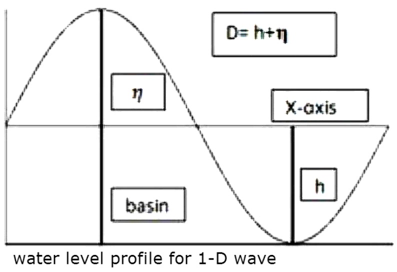

Figure 1: 1-D shallow water model

The Eqs. in (2) are coupled first order partial differential equations which can be uncoupled to produce two second order partial differential equations [4] which are given as follows

$$\begin{cases} \dfrac{\partial^2 M}{\partial t^2} = g(\eta + h)\dfrac{\partial^2 M}{\partial x^2} - \dfrac{1}{(\eta + h)}\dfrac{\partial M}{\partial x}\dfrac{\partial M}{\partial t} \\ \\ \dfrac{\partial^2 \eta}{\partial t^2} = g(\eta + h)\dfrac{\partial^2 \eta}{\partial x^2} + g\left(\dfrac{\partial \eta}{\partial x}\right)^2 + g\dfrac{\partial h}{\partial x}\dfrac{\partial \eta}{\partial x} \end{cases} \tag{3}$$

3 Finite difference method (FDM) for solving one dimensional SWEs

In this report we have used FDM for solution of one dimensional SWEs.

3.1 Intoduction to finite difference method (FDM)

Finite-difference methods are numerical methods for approximating the solutions to differential equations using finite difference equations to approximate derivatives.

From Taylors Series expansion, we have

$$\Phi(x+\Delta x) = \Phi(x) + \frac{\partial \Phi}{\partial x}(\Delta x) + \frac{1}{2!}\frac{\partial^2 \Phi}{\partial x^2}(\Delta x)^2 +, ..., + \frac{1}{n!}\frac{\partial^n \Phi}{\partial x^n}(\Delta x)^n. \tag{4}$$

The grid generation is well known in FDM and so has been depicted from Fig. 2

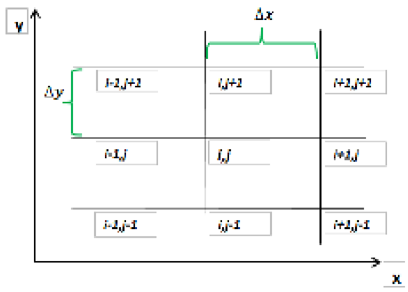

Figure 2: Grid generation in FDM

In view of Fig. 2 and Eq. (4) we can write

$$\Phi_i^{j+1} = \Phi_i^j + \left(\frac{\partial \Phi}{\partial x}\right)_i^j \Delta x + \left(\frac{\partial^2 \Phi}{\partial x^2}\right)_i^{j+1} (1/2!)(\Delta x)^2 + \left(\frac{\partial^3 \Phi}{\partial x^3}\right)_i^j (1/3!)(\Delta x)^3 + ...$$

$$\Rightarrow \left(\frac{\partial \Phi}{\partial x}\right)_i^j = \frac{(\Phi_i^{j+1} - \Phi_i^j)}{\Delta x} - \left(\frac{\partial^2 \Phi}{\partial x^2}\right)_i^{j+1} (1/2!)\Delta x - \left(\frac{\partial^3 \Phi}{\partial x^3}\right)_i^j (1/3!)(\Delta x)^2. \tag{5}$$

As such forward difference approximation may be written as

$$\left(\frac{\partial \Phi}{\partial x}\right)_i^j = \frac{(\Phi_{i+1}^j - \Phi_i^j)}{\Delta x} + \bigcirc(\Delta x) \tag{6}$$

Similarly we have, backward difference approximation as

$$\left(\frac{\partial \Phi}{\partial x}\right)_i^j = \frac{(\Phi_i^j - \Phi_{i-1}^j)}{\Delta x} + \bigcirc(\Delta x) \tag{7}$$

Finally the central difference approximation is

$$\Phi_i^{j+1} = \Phi_i^j + \left(\frac{\partial \Phi}{\partial x}\right)_i^j \Delta x + \left(\frac{\partial^2 \Phi}{\partial x^2}\right)_i^{j+1} (1/2!)(\Delta x)^2 + \left(\frac{\partial^3 \Phi}{\partial x^3}\right)_i^j (1/3!)(\Delta x)^3 + ... \quad (8)$$

and

$$\Phi_{i-1}^j = \Phi_i^j - \left(\frac{\partial \Phi}{\partial x}\right)_i^j \Delta x + \left(\frac{\partial^2 \Phi}{\partial x^2}\right)_i^{j+1} (1/2!)(\Delta x)^2 - \left(\frac{\partial^3 \Phi}{\partial x^3}\right)_i^j (1/3!)(\Delta x)^3 + ... \quad (9)$$

Subtracting (9) from (8), we get

$$\Phi_{i+1}^j - \Phi_{i-1}^j = 2\left(\frac{\partial \Phi}{\partial x}\right)_i^j \Delta x + \bigcirc(\Delta x)^2$$

$$\Rightarrow \left(\frac{\partial \Phi}{\partial x}\right)_i^j = \frac{\left(\Phi_{i+1}^j - \Phi_{i-1}^j\right)}{2\Delta x} + \bigcirc(\Delta x)^2 \quad (10)$$

3.2 Different schemes of FDM

We have used different schemes of FDM such as Explicit, Semi-implicit, and Implicit schemes for numerical solutions of one dimensional shallow water equations.

3.3 Explicit scheme

To solve the SWEs in one dimension we discretize the first Eq. (2) with respect to both space and time. We approximate the time derivative by forward difference and space derivative by central difference, thus we have

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} &= 0 \\ \Rightarrow \left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] + \left[\frac{M_{i+1}^j - M_{i-1}^j}{2\ \Delta x}\right] &= 0 \\ \Rightarrow \left[\eta_i^{j+1} - \eta_i^j\right] &= -\frac{\Delta t}{2\ \Delta x}\left[M_{i+1}^j - M_{i-1}^j\right] \end{aligned}$$

$$\Rightarrow \eta_i^{j+1} = \eta_i^j - (c/2)\left[M_{i+1}^j - M_{i-1}^j\right] \quad (11)$$

Also from second Eq. of (2), we have

$$\frac{\partial M}{\partial t} + gD\frac{\partial \eta}{\partial x} = 0$$

$$\Rightarrow \left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right] + gD\left[\frac{\eta_{i+1}^j - \eta_{i-1}^j}{2\ \Delta x}\right] = 0$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\ (\eta_i^j + h)\left[\eta_{i+1}^j - \eta_{i-1}^j\right] \tag{12}$$

Initially we are taking the basin depth to be zero. i.e. $h = 0$. So from the above the equation we get

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\ (\eta_i^j)\left[\eta_{i+1}^j - \eta_{i-1}^j\right] \tag{13}$$

where, $c = \dfrac{\Delta t}{\Delta x}$, which is the ratio between time step and spatial step.

3.4 Semi-implicit scheme

Using this scheme of FDM we can approximate [4] the first Eq. given in (2) as follows

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0$$

$$\Rightarrow \left[\frac{\eta_i^{j+1} - \eta_{i-1}^j}{\Delta t}\right] + \left[\frac{M_{i+1}^j - M_{i-1}^j}{2\ \Delta x}\right] = 0$$

$$\Rightarrow \left[\eta_i^{j+1} - \eta_i^j\right] = -\frac{\Delta t}{2\ \Delta x}\left[M_{i+1}^j - M_{i-1}^j\right]$$

$$\Rightarrow \eta_i^{j+1} = \eta_i^j - (c/2)\left(M_{i+1}^j - M_{i-1}^j\right) \tag{14}$$

Now applying Crank-Nicolson approximation to the Eq. (14) we have

$$\left(M_{i+1}^j - M_{i-1}^j\right) = 1/2\left[(M_{i+1}^j - M_{i-1}^j) + (M_{i+1}^{j+1} - M_{i-1}^{j+1})\right]$$

$$\Rightarrow \eta_i^{j+1} = \eta_i^j - (c/4)[M_{i+1}^j - M_{i-1}^j] - (c/4)[M_{i+1}^{j+1} - M_{i-1}^{j+1}] \tag{15}$$

Again from the second Eq. of (2), we have

$$\frac{\partial M}{\partial t} + g\ D\ \frac{\partial \eta}{\partial x} = 0$$

$$\Rightarrow \left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right] + g\ D\ \left[\frac{\eta_{i+1}^j - \eta_{i-1}^j}{2\ \Delta x}\right] = 0$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\left[\eta_i^j + h\right]\left[\eta_{i+1}^j - \eta_{i-1}^j\right]$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\left[\eta_i^j\right]\left[\eta_{i+1}^j - \eta_{i-1}^j\right] \tag{16}$$

3.5 Implicit scheme

We now implement another scheme of FDM known as implicit method [4]on the one dimensional SWEs. In this method we have used Crank-Nicolson approximation, which is the average of the central differences about the point (i, j) and $(i, j+1)$. This method has a stable solution for any value of Δt and Δx . So, after applying this method in the first Eq. of (2) and solving we get

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x} = 0$$

$$\Rightarrow \frac{\eta_i^{j+1}-\eta_i^j}{\Delta t}+\frac{M_{i+1}^j-M_{i-1}^j}{2\ \Delta x}=0$$

$$\Rightarrow \left(\eta_i^{j+1}-\eta_i^j\right) = -\frac{\Delta t}{2\ \Delta x}\left(M_{i+1}^j-M_{i-1}^j\right)$$

$$\Rightarrow \eta_i^{j+1}=\eta_i^j-(c/2)\left(M_{i+1}^j-M_{i-1}^j\right) \tag{17}$$

Again applying Crank-Nicolson approximation to the above Eq. we have

$$\left(M_{i+1}^j-M_{i-1}^j\right)=1/2\left[(M_{i+1}^j-M_{i-1}^j)+(M_{i+1}^{j+1}-M_{i-1}^{j+1})\right]$$

$$\Rightarrow \eta_i^{j+1}=\eta_i^j-(c/4)(M_{i+1}^j-M_{i-1}^j)-(c/4)(M_{i+1}^{j+1}-M_{i-1}^{j+1})$$

$$\Rightarrow \eta_i^{j+1}+(c/4)(M_{i+1}^{j+1}-M_{i-1}^{j+1}) = \eta_i^j-(c/4)(M_{i+1}^j-M_{i-1}^j) \tag{18}$$

From the second Eq. of (2) now we have

$$\frac{\partial M}{\partial t}+g\ D\frac{\partial \eta}{\partial x} = 0$$

$$\Rightarrow \left[\frac{M_i^{j+1}-M_i^j}{\Delta t}\right]+gD\left[\frac{\eta_{i+1}^j-\eta_{i-1}^j}{2\ \Delta x}\right]=0$$

$$\Rightarrow M_i^{j+1}=M_i^j-(1/2)cg\left(\eta_i^j+h\right)\left(\eta_{i+1}^j-\eta_{i-1}^j\right)$$

$$\Rightarrow M_i^{j+1}+(1/4)cgh\left[\eta_{i+1}^{j+1}-\eta_{i-1}^{j+1}\right]=M_i^j-(1/2)cg\left(\eta_i^j\right)\left[\eta_{i+1}^j-\eta_{i-1}^j\right]-(1/4)cgh\left[\eta_{i+1}^j-\eta_{i-1}^j\right] \tag{19}$$

Left hand sides of Eqs. (18) and (19) are the terms of η and M at time step $j+1$. The right hand sides are at time step j. So, we can solve the above system of Eqs. by any well known method.

3.6 Tsunami wave approximation

In previous sections, we have assumed the basin depth of water to be zero. i.e. $h=0$. Now we have approximated the basin depth h by a hyperbolic tangent [4] as given below

$$h(x)=50-45\ tanh\ [\frac{(x-70)}{8}] \text{ where, 0 m} \leqslant \text{x} \leqslant 100\ m(20)$$

So, from Eq. (20) we can find the maximum and minimum value of h as 95 m and 5 m respectively.

Thus, for the explicit scheme case i.e. from Eq. (13) we have

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\ (\eta_i^j)\left[\eta_{i+1}^j - \eta_{i-1}^j\right]$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\ (\eta_i^j + h)\left[\eta_{i+1}^j - \eta_{i-1}^j\right] \quad (21)$$

and for the implicit scheme case i.e. from Eq. (19) we have

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)cg\left(\eta_i^j + h\right)\left(\eta_{i+1}^j - \eta_{i-1}^j\right)$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\ g\left(\eta_i^j\right)\left(\eta_{i+1}^j - \eta_{i-1}^j\right) - 1/2c\ g\ h\left(\eta_{i+1}^j - \eta_{i-1}^j\right)$$

$$\Rightarrow M_i^{j+1} = M_i^j - (1/2)c\, g\left(\eta_i^j\right)\left(\eta_{i+1}^j - \eta_{i-1}^j\right) - 1/4c\, g\, h\left[(\eta_{i+1}^j - \eta_{i-1}^j) + (\eta_{i+1}^{j+1} - \eta_{i-1}^{j+1})\right]$$

$$\Rightarrow M_i^{j+1} + 1/4c\, g\, h\left(\eta_{i+1}^{j+1} - \eta_{i-1}^{j+1}\right) = M_i^j - (1/2)c\, g\left(\eta_i^j\right)\left(\eta_{i+1}^j - \eta_{i-1}^j\right) - 1/4c\, g\, h\left(\eta_{i+1}^j - \eta_{i-1}^j\right) \quad (22)$$

4 Finite volume method (FVM) for solving one dimensional SWEs

In this report we have used another numerical method known as finite volume method (FVM) for solution of one dimensional SWEs.

4.1 Introduction to finite volume method (FVM)

FVM is a method of representing the partial differential equations into algebraic equations. In this method, we calculate the values at discrete places or points as in finite difference or finite element methods. Finite (control) refers here as small volume surrounding each node point on a mesh.

In this method, we integrate the given partial differential equations that contains a divergence term which can be converted to surface integral using divergence theorem. FVM is based on discretization of the integral forms of the conservation equations. Discretization is applied directly to the integral equations for small control (finite) volumes as shown in the Fig. 3 [7]

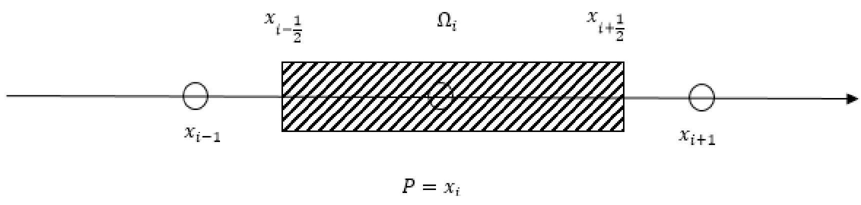

Figure 3: One dimensional control (finite) volume in FVM

In this method, instead of discretizing first, we start with the integral form of the equations.

Below we write the steps involved in FVM for solving one dimensional SWEs.

Step 1-Grid generation in FVM:

The first step in the finite volume method is grid generation i.e. by dividing the domain into discrete control volumes. The boundaries of control volumes are positioned mid-way between adjacent nodes. Thus each node is surrounded by a control volume or cell. Generally, it is better to set up control volumes near the edge of the domain in such a way that the physical boundaries coincide with the control volume boundaries. Consider a control volume whose nodal point is P and the neighbouring the nodes to the west and east, are defined as W and E respectively. The west face of the control volume is referred by w and east face by e. The distance from W to P, and P to E are given by δx_{WP} and δx_{PE} respectively as shown in the Fig. 4. Also the distance from w to P , P to e are given by δx_{wP} and δx_{Pe} respectively. The width of control volume [8] from w to e is denoted as $\Delta x = \delta x_{we}$ as shown in the fig (4).

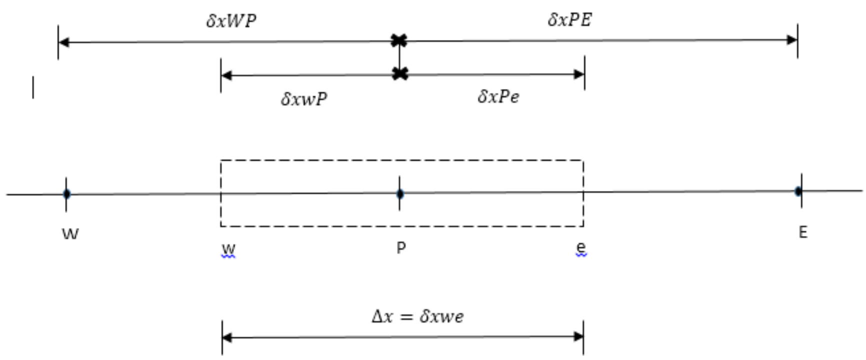

Figure 4: One dimensional grid for FVM [8]

Step 2-Discretization Concept:

The key step of FVM is the integration of the governing equation over a finite (control) volume to yield a discretised equation at its nodal point P.

Step 3- Solution of the problem:

Discretised equation must be set at each of the nodal points in order to solve a problem.The resulting system of linear algebraic equations is then solved by any well known numerical method.

4.2 Different schemes of FVM

We have used two schemes of FVM namely upwind interpolation (UI) and central difference (CD) interpolation method for solving one dimensional shallow water equations .

4.3 Upwind interpolation

It is the simplest way for approximating the partial differential equations by FVM. Here, we use the value at a neighboring grid point. We have taken the nearest upwind (upstream) grid point. Thus, taking the integral form of first Eq. of (2) into small control (finite) volumes, and then discretized in the nearest grid point. The volume integral is converted into surface integral using divergence theorem. Thus, from first Eq. (2) we have

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0 \tag{23}$$

Integrating over control (finite) volume, we get

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} \eta dx \ + \ M_e \ - \ M_w \tag{24}$$

Now we approximate the above Eq. [7]

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} \eta dx \approx \eta_P \Delta x \tag{25}$$

At the cell boundary i.e. in the east face $e = x_{i+1/2}$, the normal n is in the positive direction, so

$$M_e \approx M_P$$

Again at the cell boundary i.e. at the west face $w = x_{i-1/2}$ the normal is in the negative direction. So, taking the value at the nearest grid point in the west of the cell we have

$$M_w \approx M_W$$

So the finite volume approximation of the above Eq. (24) is

$$\frac{d}{dt}(\eta_P \Delta x) + M_P - M_W = 0$$

$$\Rightarrow \left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] + \left[\frac{M_i^j - M_{i-1}^j}{\Delta x}\right] = 0$$

$$\Rightarrow \Delta x \left(\eta_i^{j+1} - \eta_i^j\right) = \Delta t \left(M_i^j - M_{i-1}^j\right)$$

$$\Rightarrow \eta_i^{j+1} = \eta_i^j - c\left[M_i^j - M_{i-1}^j\right] \tag{26}$$

where, $c = \dfrac{\Delta t}{\Delta x}$, which is the ratio of time step and spatial step.

Again from the second Eq. of (2) we have

$$\frac{\partial M}{\partial t} + gD\frac{\partial \eta}{\partial x} = 0 \tag{27}$$

Integrating over control (finite) volume, we get

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx \ + \int_{x_{i-1/2}}^{x_{i+1/2}} g(\eta + h)\frac{\partial \eta}{\partial t} dx = 0 \tag{28}$$

where, g is the acceleration due to gravity, and h is the basin depth of water. Taking h to be zero, the Eq. becomes

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx \ + \int_{x_{i-1/2}}^{x_{i+1/2}} g(\eta)\frac{\partial \eta}{\partial t} dx = 0$$

$$\Rightarrow \frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx + g \ \int_{x_{i-1/2}}^{x_{i+1/2}} \eta\frac{\partial \eta}{\partial x} dx = 0$$

$$\Rightarrow \frac{d}{dt}(M_P \Delta x) + \frac{g}{2}\left[(\eta_e + \eta_w)\ (\eta_e - \eta_w)\right] = 0$$

$$\Rightarrow \left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right] + \frac{g}{2}\left[\frac{(\eta_p + \eta_w)(\eta_p - \eta_w)}{\Delta x}\right] = 0$$

$$\Rightarrow \left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right] + \frac{g}{2}\left[\frac{(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)}{\Delta x}\right] = 0$$

$$\Rightarrow \left[M_i^{j+1} - M_i^j\right] = \frac{\Delta t}{\Delta x}\frac{g}{2}\left[(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)\right] = 0$$

$$\Rightarrow \left[M_i^{j+1} - M_i^j\right] = \frac{c\ g}{2}\left[(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)\right] = 0$$

$$\Rightarrow M_i^{j+1} = M_i^j - \frac{cg}{2}\left[(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)\right] \tag{29}$$

where, $c = \dfrac{\Delta t}{\Delta x}$

4.4 Central difference (CD) interpolation

This approximation is based on central difference interpolation between two neighboring grid points. Using this interpolation we solve the first Eq. of (2) in the following way
Integrating over control (finite) volume of the grid points as shown in Fig. 4 we have

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} \eta dx \; + \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial M}{\partial x} dx = 0 \tag{30}$$

Approximating we have [7]

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} \eta dx \approx \eta_P \Delta x \tag{31}$$

Now, the x derivatives are approximated by central difference interpolation. Accordingly we have the following

$$\begin{aligned}
&\left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] \Delta x + (M)_e - (M)_w = 0 \\
\Rightarrow &\left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] \Delta x + [(M)_E - (M)_W] = 0 \\
\Rightarrow &\left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] + \left[\frac{(M)_E - (M)_W}{\Delta x}\right] = 0 \\
\Rightarrow &\left[\frac{\eta_i^{j+1} - \eta_i^j}{\Delta t}\right] + \left[\frac{M_{i+1}^j - M_{i-1}^j}{\Delta x}\right] = 0 \\
\Rightarrow &\, \eta_i^{j+1} = \eta_i^j - c\left[M_{i+1}^j - M_{i-1}^j\right]
\end{aligned} \tag{32}$$

Again from the second Eq. of (2) we have

$$\frac{\partial M}{\partial t} + gD\frac{\partial \eta}{\partial x} = 0 \tag{33}$$

Integrating over control (finite) volume, we get

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx \; + \int_{x_{i-1/2}}^{x_{i+1/2}} g(\eta + h)\frac{\partial \eta}{\partial x} dx = 0 \tag{34}$$

Assume h to be zero, the Eq. becomes

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx + \; g \int_{x_{i-1/2}}^{x_{i+1/2}} \eta \frac{\partial \eta}{\partial x} \Delta x = 0 \tag{35}$$

Also,
$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} M dx \approx M_P \Delta x \tag{36}$$

The x derivatives are approximated by central difference interpolation. Thus, we have the following Eqs. as

$$\frac{d}{dt}\left[M_P(\Delta x)\right] + g\ I = 0 \tag{37}$$

where,
$$I = \int_{x_{i-1/2}}^{x_{i+1/2}} \eta \frac{\partial \eta}{\partial x} dx \tag{38}$$

Now, solving I near the neighbouring grid points we have

$$\begin{aligned} I &= \frac{1}{2}\left[(\eta_E + \eta_W)(\eta_E - \eta_W)\right] \\ \Rightarrow I &= \frac{1}{2}\left[(\eta_{i+1}^j + \eta_{i-1}^j)(\eta_{i+1}^j - \eta_{i-1}^j)\right] \end{aligned} \tag{39}$$

Putting the value of I in the Eq. (37) we get

$$\begin{aligned} &\frac{d}{dt}(M_P \Delta x) + g\ \frac{1}{2}\left[(\eta_{i+1}^j + \eta_{i-1}^j)(\eta_{i+1}^j - \eta_{i-1}^j)\right] = 0 \\ \Rightarrow &\left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right]\Delta x + \frac{g}{2}\left[(\eta_{i+1}^j + \eta_{i-1}^j)(\eta_{i+1}^j - \eta_{i-1}^j)\right] = 0 \\ \Rightarrow &M_i^{j+1} - M_i^j + \frac{\Delta t}{\Delta x}\ \frac{g}{2}\left[(\eta_{i+1}^j + \eta_{i-1}^j)(\eta_{i+1}^j - \eta_{i-1}^j)\right] = 0 \\ \Rightarrow &M_i^{j+1} = M_i^j - \frac{cg}{2}\left[\left(\eta_{i+1}^j + \eta_{i-1}^j\right)\left(\eta_{i+1}^j - \eta_{i-1}^j\right)\right] \end{aligned} \tag{40}$$

The values of M in the left hand side of Eq. (40) represents the values at time step $j+1$ and the right hand side term of Eq. represents the values at time step j. Thus we can solve Eqs. (32) and (40) for evaluating the values of M and η by an iterative method.

4.5 Tsunami wave approximation

Initially, in FVM we have assumed the basin depth of water to be zero. i.e. $h = 0$. Now we have approximated the basin depth h by a hyperbolic tangent (as taken in FDM)

$$h(x) = 50 - 45\ tanh\ [\frac{(x-70)}{8}] \text{ where, } 0\ m\ \leqslant \text{x} \leqslant 100\ m (41)$$

Thus, from Eq. (28) we have

$$\frac{d}{dt}\int_{x_{i-1/2}}^{x_{i+1/2}} Mdx\ + g\ \int_{x_{i-1/2}}^{x_{i+1/2}} (\eta + h)\frac{\partial \eta}{\partial x}dx = 0 \tag{42}$$

Let us assume,

$$I = \int_{x_{i-1/2}}^{x_{i+1/2}} (\eta + h)\frac{\partial \eta}{\partial x}$$
$$\Rightarrow I = \int_{x_{i-1/2}}^{x_{i+1/2}} \eta\frac{\partial \eta}{\partial x} + \int_{x_{i-1/2}}^{x_{i+1/2}} h\frac{\partial \eta}{\partial x} \tag{43}$$

Now,

$$\int_{x_{i-1/2}}^{x_{i+1/2}} \eta\frac{\partial \eta}{\partial x} = \frac{1}{2}\left[(\eta_e + \eta_w)\ (\eta_e - \eta_w)\right] \tag{44}$$

and,

$$\int_{x_{i-1/2}}^{x_{i+1/2}} h\frac{\partial \eta}{\partial x} = h\,(\eta_e - \eta_w) \tag{45}$$

So,

$$I = \frac{1}{2}\left[(\eta_e + \eta_w)\ (\eta_e - \eta_w)\right] + h\,(\eta_e - \eta_w) \tag{46}$$

Now, putting the value of I in Eq. (42) and solving we get,

$$\frac{d}{dt}(M_P\Delta x) + \frac{g}{2}\left[(\eta_e + \eta_w)\ (\eta_e - \eta_w)\right] + gh\,(\eta_e - \eta_w) = 0$$
$$\Rightarrow \left[\frac{M_i^{j+1} - M_i^j}{\Delta t}\right] + \frac{g}{2\Delta x}\left[(\eta_p + \eta_w)(\eta_p - \eta_w) + gh(\eta_p - \eta_w)\right] = 0$$
$$\Rightarrow \left[M_i^{j+1} - M_i^j\right] + \frac{cg}{2}\left[(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)\right] +\ gh\left[\eta_i^j - \eta_{i-1}^j\right] = 0$$
$$\Rightarrow M_i^{j+1} = M_i^j - \frac{cg}{2}\left[(\eta_i^j + \eta_{i-1}^j)(\eta_i^j - \eta_{i-1}^j)\right] - gh\left[\eta_i^j - \eta_{i-1}^j\right] (47)$$

4.6 Interval finite volume method (IFVM)

The uncertain values occurred in practical cases (such as errors in experimental data, and partial or imperfect knowledge of the parameters) may be handled by taking the uncertainty as interval sense. So to compute these uncertainties we need interval arithmetic. Let us consider the uncertain values of a parameter η in interval form and the same may be written in the following way

$$[\ \underline{x}\ ,\overline{x}\ \ ] = [\ x \mid x \in R,\ \underline{x} \leq x \leq \overline{x}\ \ ] \tag{48}$$

where, $[\ \underline{x}\ ]$ and $[\ \overline{y}\ ]$ are lower and upper values of the interval respectively. Let us assume two intervals $[\ \underline{x}\ ,\overline{x}\ \ ]$ and $\left[\ \underline{y}\ ,\overline{y}\ \ \right]$ then we have

$$\begin{aligned}
[\ \underline{x}\ ,\overline{x}\ \ ] + \left[\ \underline{y}\ ,\overline{y}\ \ \right] &= \left[\ \underline{x}+\underline{y}\ ,\overline{x}+\overline{y}\ \ \right] \\
[\ \underline{x}\ ,\overline{x}\ \ ] - \left[\ \underline{y}\ ,\overline{y}\ \ \right] &= \left[\ \underline{x}-\overline{y}\ ,\overline{x}-\underline{y}\ \ \right]
\end{aligned} \tag{49}$$

Upwind Interpolation IFVM:

Now applying the IFVM in Eqs. (26) and (29) we get

$$\begin{aligned}
[\ \underline{M},\overline{M}\ ]_i^{j+1} &= [\ \underline{M},\overline{M}\ ]_i^j - (cg/2)\left[\left[\ [\ \underline{\eta},\overline{\eta}\ ]_i^j + [\ \underline{\eta},\overline{\eta}\ ]_{i-1}^j\right]\left[[\ \underline{\eta},\overline{\eta}\ ]_i^j - [\ \underline{\eta},\overline{\eta}]_{i-1}^j\right]\right] \\
[\ \underline{\eta},\overline{\eta}\ ]_i^{j+1} &= [\ \underline{\eta},\overline{\eta}\ ]_i^j - c[\ \underline{M},\overline{M}\ ]_i^j - [\ \underline{M},\overline{M}\ ]_{i-1}^j
\end{aligned} \tag{50}$$

Rearranging the above equations we get a set of four Eqs. as follows

$$\begin{aligned}
[\ \underline{M}\ ]_i^{j+1} &= [\ \underline{M}\ ]_i^j - (cg/2)\ [\ \underline{\eta}\ ]_i^j + [\ \underline{\eta}\ ]_{i-1}^j\ [\ \underline{\eta}\ ]_i^j - [\ \overline{\eta}\ ]_{i-1}^j \\
[\ \overline{M}\ ]_i^{j+1} &= [\ \overline{M}\ ]_i^j - (cg/2)\ [\ \overline{\eta}\ ]_i^j + [\ \overline{\eta}\ ]_{i-1}^j\ [\ \overline{\eta}\ ]_i^j - [\ \underline{\eta}\ ]_{i-1}^j \\
[\ \underline{\eta}\ ]_i^{j+1} &= [\ \underline{\eta}\ ]_i^j - c\ [\ \underline{M}\ ]_i^j - [\ \overline{M}\ ]_{i-1}^j \\
[\ \overline{\eta}\ ]_i^{j+1} &= [\ \overline{\eta}\ ]_i^j - c\ [\ \overline{M}\ ]_i^j - [\ \underline{M}\ ]_{i-1}^j
\end{aligned} \tag{51}$$

Central Difference Interpolation IFVM:

Applying the IFVM the Eqs. (32) and (40) we have

$$\begin{aligned}
[\ \underline{M},\overline{M}\ ]_i^{j+1} &= [\ \underline{M},\overline{M}\ ]_i^j - (cg/2)\left[\left[[\ \underline{\eta},\overline{\eta}\ ]_{i+1}^j + [\ \underline{\eta},\overline{\eta}\ ]_{i-1}^j\right]\left[[\ \underline{\eta},\overline{\eta}\ ]_{i+1}^j - [\ \underline{\eta},\overline{\eta}]_{i-1}^j\right]\right] \\
[\ \underline{\eta},\overline{\eta}\ ]_i^{j+1} &= [\ \underline{\eta},\overline{\eta}\ ]_i^j - c\left[[\ \underline{M},\overline{M}\ ]_{i+1}^j - [\ \underline{M},\overline{M}\ ]_{i-1}^j\right]
\end{aligned} \tag{52}$$

Rearranging the above Eqs. one can get a set of four Eqs. as below

$$\begin{aligned}
[\ \underline{M}\ ]_i^{j+1} &= [\ \underline{M}\ ]_i^j - (cg/2)\left[\left[[\ \underline{\eta}]_{i+1}^j + [\ \underline{\eta}\ ]_{i-1}^j\right]\left[[\ \underline{\eta}\ ]_{i+1}^j - [\ \overline{\eta}\ ]_{i-1}^j\right]\right] \\
[\ \overline{M}\ ]_i^{j+1} &= [\ \overline{M}\ ]_i^j - (cg/2)\left[\left[[\ \overline{\eta}\ ]_{i+1}^j + [\ \overline{\eta}\ ]_{i-1}^j\right]\left[[\ \overline{\eta}\ ]_{i+1}^j - [\ \underline{\eta}\ ]_{i-1}^j\right]\right] \\
[\ \underline{\eta}\ ]_i^{j+1} &= [\ \underline{M}\ ]_i^j - c\left[[\ \underline{M}\ ]_{i+1}^j - [\ \overline{M}\ ]_{i-1}^j\right] \\
[\ \overline{\eta}\ ]_i^{j+1} &= [\ \overline{M}\ ]_i^j - c\left[[\ \overline{M}\ ]_{i+1}^j - [\ \underline{M}\ ]_{i-1}^j\right]
\end{aligned} \tag{53}$$

5 Numerical results

As mentioned earlier we have used finite difference method (FDM) and finite volume method (FVM) for solution of one dimensional SWEs. Moreover interval finite volume method (IFVM) has also been developed to solve the SWEs in uncertain environment.

5.1 Numerical results using FDM

For one dimensional shallow water equations we have shown the numerical results for different schemes of FDM. We have used [4] grid size $\Delta x = \left(\frac{1}{8}\right) m$ for space and $\Delta t = \left(\frac{1}{3000}\right) s$ for time step.

The boundary conditions [4] have been taken as

at $x = 0$ and $x = L$ $M = 0$.

Also $\eta(0, j) = \eta(0, j)$; $\eta(L - 1, j) = \eta(L, j)$

The initial conditions are taken as

at $t = 0$, assuming the initial velocity of water is zero, i.e. water is at rest position at $t = 0$ and therefore $M = 0$. First we take the basin depth of water to be zero i.e. $h = 0$. Another assumption is, the maximum and minimum values of η are 18 m and 20 m respectively.

And the corresponding MATLAB program has been developed to compute the result for the behavoiur water surface elevation (i.e. η) with distance x is shown in Fig. 5 at ($h = 0$, $t = 0$)

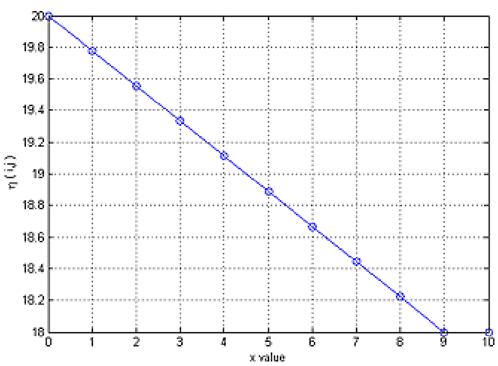

Figure 5: At $t = 0s$, graph of η with distance x

We have shown the graphs for variation of water surface elevation η with different value of time t in case of explicit scheme of FDM in Fig.6 to 10 for different value of t.

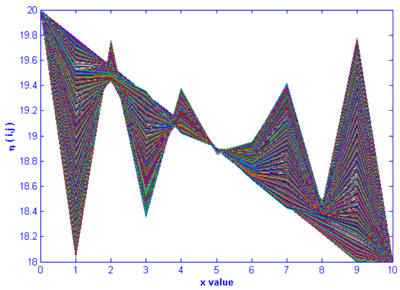

Figure 6: At time $t = 0.1\ s$

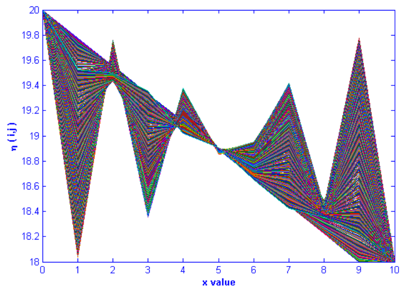

Figure 7: At time $t = 0.16667\ s$

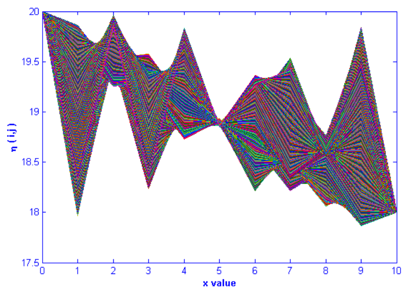

Figure 8: At time $t = 0.3333\ s$

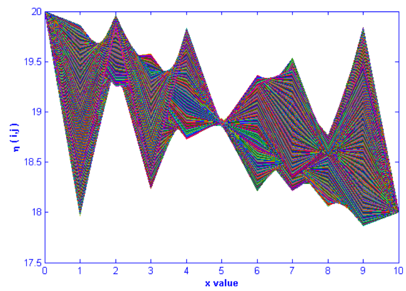

Figure 9: At time $t = 1\ s$

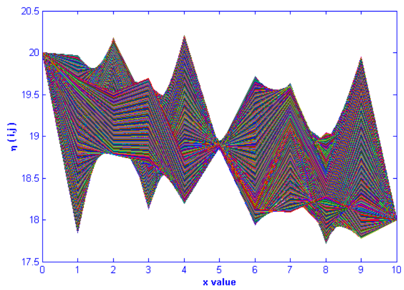

Figure 10: At time $t = 1.6666\ s$

Again we have shown the plots for variation of water surface elevation η with different values of time t in case of implicit scheme of FDM in Fig.11 to 15.

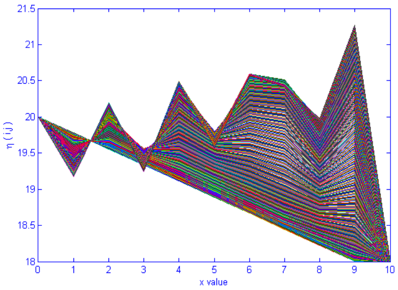

Figure 11: At time $t = 0.1\ s$

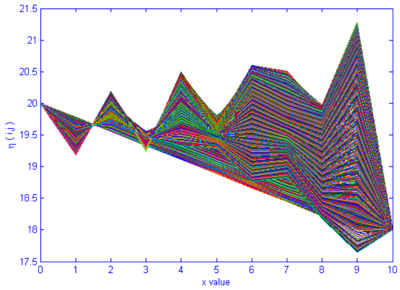

Figure 12: At time $t = .16667\ s$

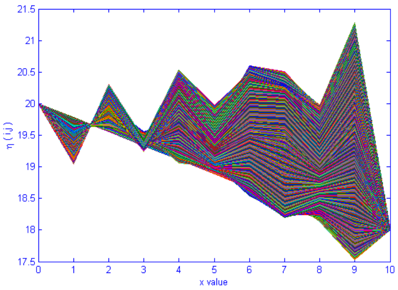

Figure 13: At time $t = 0.333\ s$

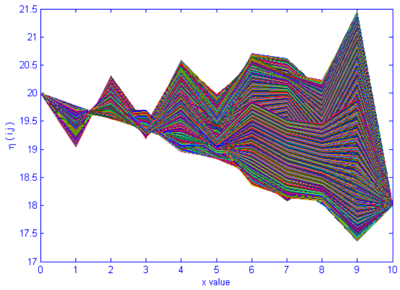

Figure 14: At time $t = 1\ s$

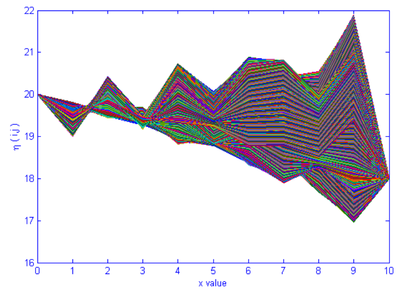

Figure 15: At time $t = 1.6667\ s$

Initially we have assumed the basin depth of water i.e. $h = 0$. Now from Eq. (20) we take the minimum and maximum value of basin depth as $h = 5\ m$ and $h = 95\ m$ respectively. Accordingly we have shown the behaviour of η with distance in the Fig. 16 and 17 for both the cases.

The behaviour of water surface elevation η with distance x when the basin depths are $85m$ $90m$ and $95m$ is depicted in Fig. 18.

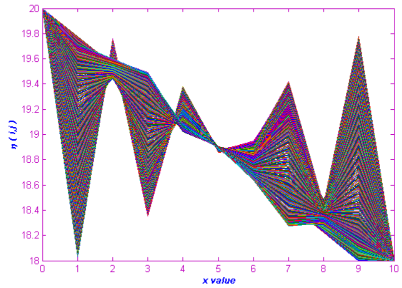

Figure 16: At $t = 0.1s$, graph of η with distance x when $h = 5m$

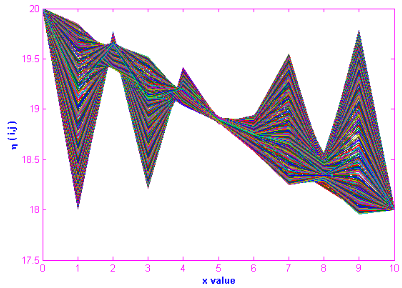

Figure 17: At $t = 0.1s$, graph of η with distance x when $h = 95m$

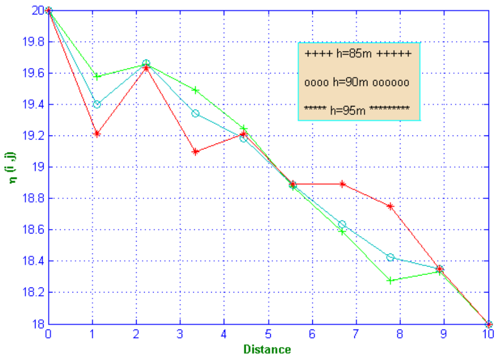

Figure 18: graph for $h = 85m,\ 90m,\ 95m$

5.2 Numerical results using FVM and IFVM

For one dimensional shallow water equations we have shown the numerical results for different schemes of FVM. Grid size $\Delta x = \left(\frac{1}{8}\right) m$ and time step size $\Delta t = \left(\frac{1}{3000}\right) s$ have been considered.

The boundary conditions [4] have been taken similar to FDM as

at $x = 0$ and $x = L$ $M = 0$.

and $\eta(0, j) = \eta(0, j)$; $\eta(L - 1, j) = \eta(L, j)$

The initial condition has been taken as
at $t = 0$, assuming the initial velocity of water is zero, i.e water is at rest position at $t = 0$ and therefore $M = 0$

Similar to the case of FDM we first take the basin depth of water to be zero i.e the value of $h = 0$. Another assumption is the maximum and minimum values of η are 18 m and 20 m respectively.

Thus in case of FVM when the initial velocity of water is zero i.e at $t = 0$ the variation of water surface elevation η with different value of x is shown in Fig. 19 using MATLAB.

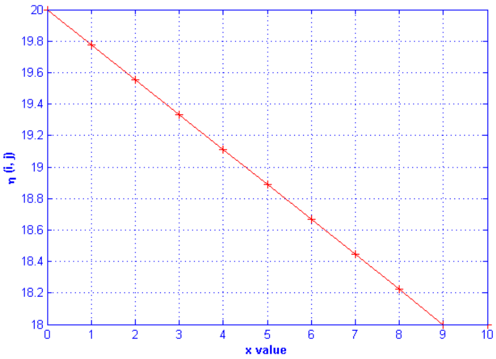

Figure 19: At $t = 0s$, graph of η with distance x

The plots for variation of water surface elevation η with different value of time t in case of upwind interpolation of FVM has been shown in Fig. 20 and 21.

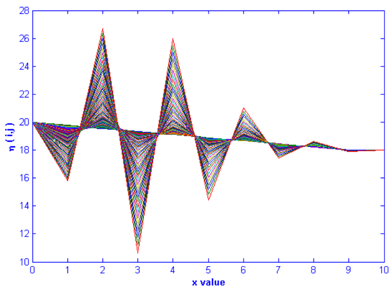

Figure 20: At time $t = 0.3333\ s$

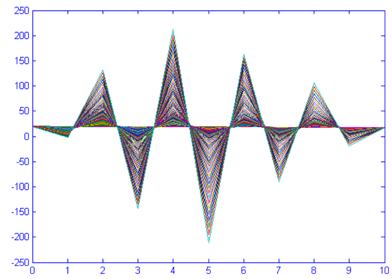

Figure 21: At time $t = 0.5\ s$

Similarly the plots for variation of water surface elevation η with different values of time t in case of central difference interpolation of FVM has been shown in Fig. 22 to 27.

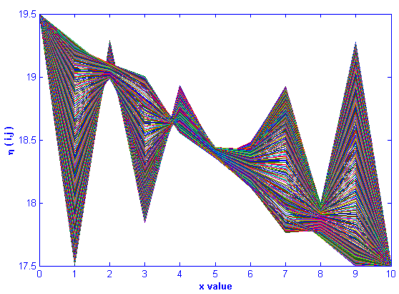

Figure 22: At time $t = 0.1$

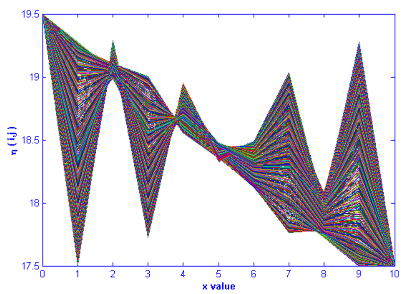

Figure 23: At time $t = 0.16667$

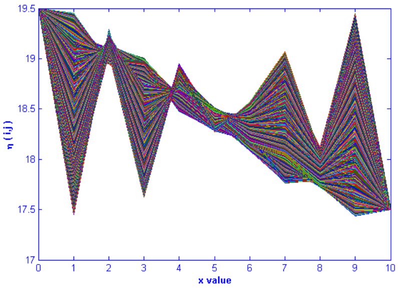

Figure 24: At time $t = 0.3333$

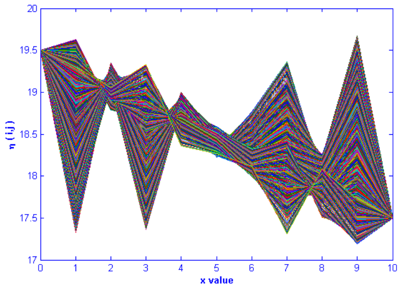

Figure 25: At time $t = 1$

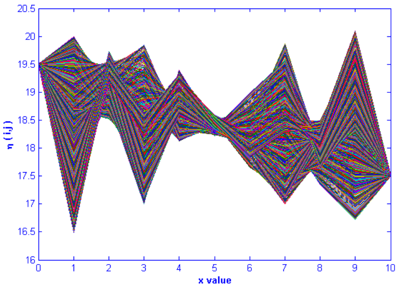

Figure 26: At time $t = 1$

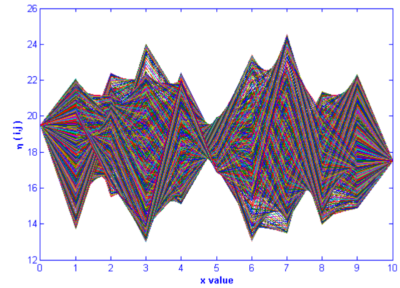

Figure 27: At time $t = 1.6666$

As per Eq. (20) the minimum and maximum value of basin depth i.e $h = 5\ m$ and $h = 95\ m$ respectively have been considered. The behaviour of η with distance is cited in Fig. 28 and Fig 29.

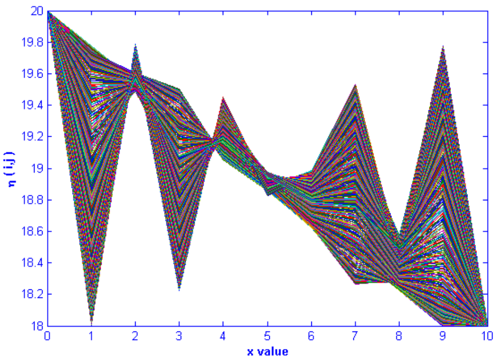

Figure 28: At $t = 0.1s$, graph of η with distance x when $h = 5m$

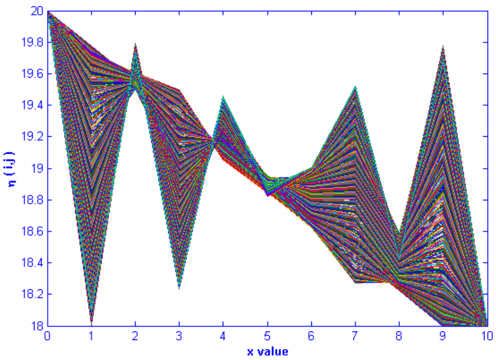

Figure 29: At $t = 0.1s$, graph of η with distance x when $h = 95m$

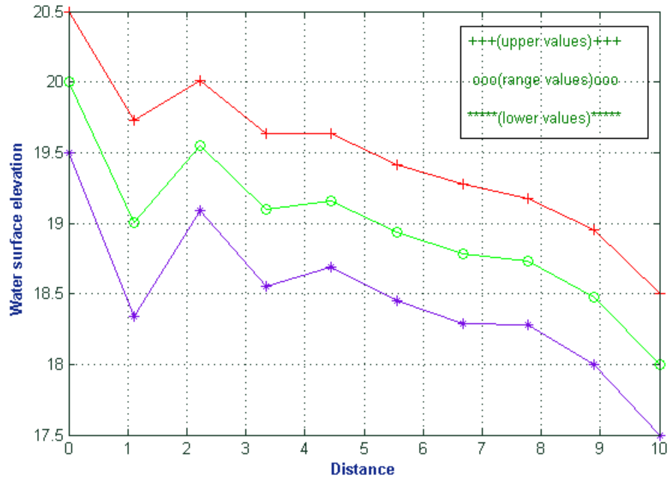

Figure 30: graph of η with distance x

Numerical results using IFVM

In this case the values of η and M, are in an interval as such the maximum and minimum values of η are taken in the interval as [17.5 18.5] and [19.5 20.5] respectively. Also, the value of M in the interval has been as [0, 0.5].

After taking the above values in the interval we have investigated the variation of water surface elevation η with distance x with upper interval, lower interval and crisp value which is shown in Fig. 30.

6 Comparison between FDM and FVM, conclusion and future directions

A comparative study of numerical results of FDM and FVM for one dimensional shallow water equations have been presented hare.

After investigating the numerical methods FDM and FVM for solution of one dimensional shallow water equation we have compared the values for water surface elevation η with distance x and is shown in the Fig. 31 and 32. Corresponding comparison tables for both the methods with different time are given in Tables 1 and 2.

Table 1: *At time* $t = 0.1$ *comparision of FDM with FVM for the value of water surface elevation*

S.no	FDM	FVM
1	20.0000	20.0000
2	19.1820	19.6872
3	20.1911	19.5149
4	19.1784	19.3509
5	20.4563	19.1330
6	19.5764	18.8960
7	20.5701	18.6437
8	20.4156	18.4385
9	19.9754	18.2493
10	18.0000	18.0000

Table 2: *At time* $t = 0.1$ *for FVM and at time* $t = .1667$ *for FDM the value of water surface elevation*

S.no	FDM	FVM
1	20.0000	20.0000
2	19.5752	19.6872
3	19.7183	19.5149
4	19.4802	19.3509
5	19.3298	19.1330
6	19.2880	18.8960
7	18.7672	18.6437
8	18.8293	18.4385
9	18.1797	18.2493
10	18.0000	18.0000

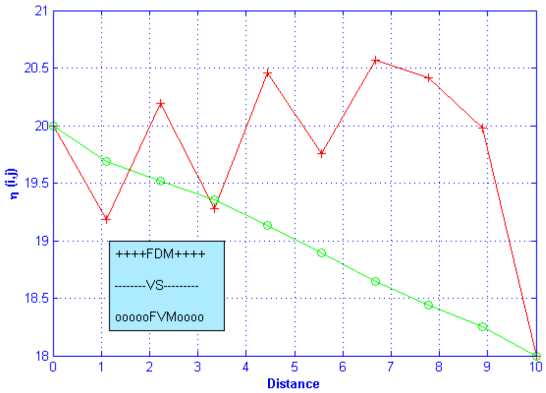

Figure 31: At time $t = 0.1\ s$

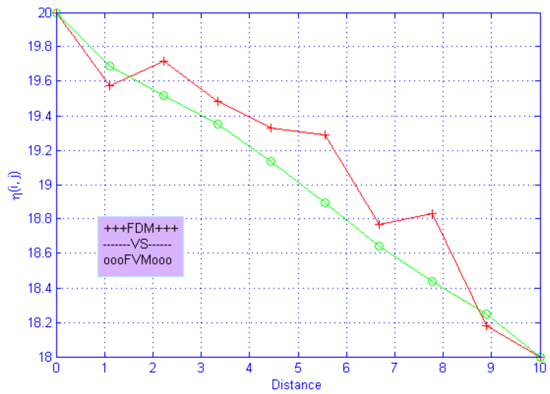

Figure 32: At time $t = 0.1\ s$ for FVM and $t = 0.1667s$ for FDM

Conclusion

Although the results for one dimensional SWEs using finite volume schemes coincide with the simple schemes of finite difference but advantage of FVM is that of the meshing scheme. The unique character of finite volume schemes usually appear when multidimensional problems are solved using unstructured grids.

The most important feature of FVM is that the method is conservative because the flux entering a given volume is identical to that leaving the adjacent volume and it is used only when the equations are based on conservation of physical laws. Another advantage of the FVM is that it is easily formulated to allow for unstructured meshes. The method is widely used in computational fluid dynamics (CFD).
Also, in FDM the values of the dependent variables are stored at the nodes only. In FEM these values are stored at the element nodes. But in FVM, the values of the dependent variables are stored at the centre of the control volume. In case of FDM and FEM, conservation of mass, momentum, energy are not ensured at each cell/control volume. But this is true for FVM. It may be worth mentioning that FVM takesless time in computation because it converges with less number of control volumes.

Future Direction

This investigation gives a new idea of the Interval FVM through SWEs and this can very well be used in future research for better results for other equations obtained from different applications. The idea may easily be extended to other structured as well as unstructured problems with various complicating effects. Although this require more complex forms of interval computation to handle the corresponding problem.

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